MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2016/2017

DEM5028 – ENGINEERING MATHEMATICS 2

(Group E15)

29 MAY 2017
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **6** pages (**4** pages with **4** questions and **2** pages for appendix).

2. Answer **ALL** questions. All necessary working steps must be shown.

3. Write all your answers in the answer booklet provided.

Please answer <u>ALL</u> questions and show the necessary working. Total mark is equal to 100.

Question 1

a. Find the volume of the solid generated when the region bounded by $y = x^2 - 1$, $y = 0$ and $x = 2$ is rotated about y-axis.

[5 marks]

b. Find the solution of the linear differential equation $\frac{dy}{dx} = -2xy + x$

[4 marks]

c. Evaluate $\iint_B x \cos xy dA$ where $B = \left\{(x, y) \mid \frac{\pi}{2} \le x \le \pi, 1 \le y \le 2,\right\}$

[5 marks]

d. Find the mass and centre of mass of a lamina with density $\rho(x, y) = 7$, which occupies a region between $x = y^2$, $x = y + 2$, $y = -1$ and $y = 2$.

[11 marks]

[Total = 25 marks]

Continued...

Question 2

a. Determine whether the sequence $\left\{\frac{n-1}{2n^2}\right\}$ converges or diverges?

[3 marks]

b. Test the following series for convergence or divergence. (Must state the test used)

i) $\sum_{n=1}^{\infty}\left(-\frac{3}{2}\right)^{n+1}$ [3 marks]

ii) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n+1}$ [4 marks]

c. Consider the power series $\sum_{n=1}^{\infty}\frac{n(x-1)^n}{2^n}$, find its radius and interval of convergence.

[10 marks]

d. Find the Maclaurin Series for $f(x)=e^{3x}$ in sigma notation.

[5 marks]

[Total = 25 marks]

Continued...

Question 3

a. Consider vector equations $3\mathbf{u} + 2\mathbf{v} = \langle 9, -4 \rangle$ and $\mathbf{u} - 3\mathbf{v} = \langle -8, -5 \rangle$, find vector **u** and **v.** [Hint: solve using simultaneous equations].

[7 marks]

b. Find the area of parallelogram determined by $\mathbf{a} = 3\mathbf{i} - 4\mathbf{j} - \mathbf{k}$ and $\mathbf{b} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$.

[6 marks]

c. Find the symmetric and parametric equations of the line that goes through the points $P(1, 2, 4)$ and $Q(3, -1, 6)$.

[7 marks]

d. Find the equation of plane through the point $(1, 2, -3)$ and perpendicular to line $x = 1 + 2t, y = 2 + t, z = -3 - 5t$

[5 marks]

[Total = 25 marks]

Continued...

Question 4

a. Find all the first partial derivatives for $f(x,y,z)=\dfrac{x+y^2}{z}$.

[3 marks]

b. Use the Chain Rule to find $\dfrac{\partial w}{\partial t}$ if $w=x^2+y^2+z^2$, $x=e^t\cos t$, $y=e^t\sin t$ and $z=e^t$.

[8 marks]

c. Given $f(x,y)=-x^3-y^2+12x-6y+7$.

i) Find the critical point(s) of the function.

[6 marks]

ii) Determine whether the critical point(s) is a maximum, minimum or saddle point.

[8 marks]

[Total = 25 marks]

End of Page.

APPENDIX I: Formulae

Integration of common functions

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \qquad \int \frac{1}{x} dx = \ln|x| + C \qquad \int e^x dx = e^x + C \qquad \int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int \sin x dx = -\cos x + C \qquad \int \cos x\, dx = \sin x + C \qquad \int \sec^2 x\, dx = \tan x + C \qquad \int \sec x \tan x dx = \sec x + C$$

$$\int \csc^2 x dx = -\cot x + C \qquad \int \csc x \cot x dx = -\csc x + C$$

Inverse Trigonometry

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$\int \frac{1}{x^2+1} dx = \tan^{-1} x + C$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \cot^2 x = \csc^2 x$$

$$1 + \tan^2 x = \sec^2 x$$

Integration by parts

$$\int u dv = uv - \int v du$$

Areas Between Curves

$$A = \int_a^b [f(x) - g(x)] dx$$

$$A = \int_c^d [w(y) - v(y)] dy$$

Volume by Washer

$$V = \int_a^b \pi\left(\left[f(x)\right]^2 - \left[g(x)\right]^2\right) dx$$

$$V = \int_c^d \pi\left(\left[w(y)\right]^2 - \left[v(y)\right]^2\right) dy$$

Volume by Cylindrical Shells

$$V = \int_a^b 2\pi x\left(f(x) - g(x)\right) dx$$

$$V = \int_c^d 2\pi y\left(w(y) - v(y)\right) dy$$

Linear Differential Equations:

$$\frac{dy}{dx} + p(x)y = q(x);\ \mu y = \int \mu q(x) dx \Rightarrow y = \frac{1}{\mu}\int \mu q(x) dx, \qquad \text{where } \mu = e^{\int p(x) dx}$$

Divergence Test	If $\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ diverges.
p-series	The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if p > 1 and divergent if $p \leq 1$.
Limit Comparison Test	Let $\sum a_n$ and $\sum b_n$ be series with positive terms such that $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ If $0 < c < \infty$, then both series converge or both diverge.
Alternating Series Test	If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + \ldots \qquad b_n > 0$ Satisfies : i. $b_{n+1} \leq b_n$ for all n ii. $\lim_{n\to\infty} b_n = 0$ then the series is convergent
Ratio Test	Let $\sum a_n$ be a series with nonzero terms such that $L = \lim_{n\to\infty} \frac{\lvert a_{n+1}\rvert}{\lvert a_n\rvert}$ a. Series converges absolutely if $L < 1$ b. Series diverges if $L > 1$ or $L = \infty$ c. No conclusion if $L = 1$

Taylor series of the function f at a (or about a or centered at a).

$$f(x)=\sum \frac{f^{(n)}(a)}{n!}(x-a)^n \quad =f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+\ldots$$

Vector

The length of the vector $\mathbf{a} =< a_1, a_2, a_3 >$ is $|a|=\sqrt{a_1{}^2+a_2{}^2+a_3{}^2}$.

If θ is the angle between the vector **a** and **b**, then $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ & $|\mathbf{a} \times \mathbf{b}|=|\mathbf{a}||\mathbf{b}| \sin\theta$

Cross Product $\mathbf{a} \times \mathbf{b}=\begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}=\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} i-\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} j+\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} k$

Equation of Line

Vector equation: $r = r_0 + tv$ Parametric equation: $x=x_0+at$, $y=y_0+bt$, $z=z_0+ct$

Equation of Plane $< a,b,c >< x-x_0, y-y_0, z-z_0 >=0$

The Chain Rule

Suppose that $z = f(x, y)$, where $x = g(t)$ and $y = h(t)$ $\Rightarrow$ $\frac{dz}{dt}=\frac{\partial f}{\partial x}\frac{dx}{dt}+\frac{\partial f}{\partial y}\frac{dy}{dt}$

Second Derivatives Test

Suppose that $f_x(a,b)=0$ and $f_y(a,b)=0$ [that is, (a, b) is a critical point of f]. Let

$D = D(a,b) = f_{xx}(a,b)f_{yy}(a,b)-\left[f_{xy}(a,b)\right]^2$

a. If $D > 0$ and $f_{xx}(a,b)>0$, then $f(a, b)$ is a local minimum.
b. If $D > 0$ and $f_{xx}(a,b)<0$, then $f(a, b)$ is a local maximum
c. If $D < 0$, then $f(a, b)$ is a saddle point.

Moments and Centers of Mass

The **moment** about the ***x*-axis:** $M_x=\iint_D y\rho(x,y)dA$

The **moment** about the **y-axis:** $M_y=\iint_D x\rho(x,y)dA$

The coordinates $(\bar{x}, \bar{y})$ of the center of mass:

$\bar{x}=\frac{M_y}{m}=\frac{1}{m}\iint_D x\rho(x,y)dA$ $\bar{y}=\frac{M_x}{m}=\frac{1}{m}\iint_D y\rho(x,y)dA$

Where the mass: $m=\iint_D \rho(x,y)dA$

Triple Integrals: $\iiint_B f(x,y,z)dV=\int_r^s\int_c^d\int_a^b f(x,y,z)dxdydz$